Orbitally Excited Baryons in Large N_c QCD*

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We present a model-independent analysis of the mass spectrum of nonstrange $\ell = 1$ baryons in large N_c QCD. The $1/N_c$ expansion is used to select and order a basis of effective operators that spans the nine observables (seven masses and two mixing angles). Comparison to the data provides support for the validity of the $1/N_c$ expansion, but also reveals that only a few nontrivial operators are strongly preferred. We show that our results have a consistent interpretation in a constituent quark model with pseudoscalar meson exchange interactions.

1. Introduction

QCD admits a useful and elegant expansion in powers of $1/N_c$, where N_c is the number of colors [1]. This expansion has been utilized successfully in baryon effective field theories to isolate the leading and subleading contributions to a variety of physical observables [2].

Here we study the mass spectrum of the nonstrange, $\ell = 1$ baryons (associated with the SU(6) **70**-plet for $N_c = 3$) in a large-N_c effective theory [3,4]. We describe the states as a symmetrized "core" of $(N_c - 1)$ quarks in the ground state plus one excited quark in a relative P state. "Quarks" in the effective theory refer to eigenstates of the spin-flavor-orbit group, SU(6) $\times$ O(3), such that an appropriately symmetrized collection of N_c of them have the quantum numbers of the physical baryons. Baryon wave functions are antisymmetric in color and symmetric in the spin-flavor-orbit indices of the quark fields. While this construction assures that we obtain states with the correct total quantum numbers, we do not assume that SU(6) is an approximate symmetry of the effective Lagrangian. Rather, we parameterize the most general way in which spin and flavor symmetries are broken by introducing a complete set of quark operators that act on the baryon states. Matrix elements of these operators are hierarchical in $1/N_c$, so that predictivity can be obtained without recourse to ad hoc phenomenological assumptions.

The nonstrange **70**-plet states which we consider in this analysis consist of two isospin-3/2 states, $\Delta_{1/2}$ and $\Delta_{3/2}$, and five isospin-1/2 states, $N_{1/2}$, $N'_{1/2}$, $N_{3/2}$, $N'_{3/2}$, and $N'_{5/2}$. The subscript indicates total baryon spin; unprimed states states have quark spin 1/2 and primed states have quark spin 3/2. These quantum numbers imply that two mixing

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angles, θ_{N1} and θ_{N3}, are necessary to specify the total angular momentum 1/2 and 3/2 nucleon mass eigenstates, respectively. Definitions for these angles can be found in Ref. [5].

2. Operators Analysis

To parameterize the complete breaking of SU(6)×O(3), it is natural to write all possible mass operators in terms of the generators of this group. The generators of orbital angular momentum are denoted by ℓ^i, while S^i, T^a, and G^{ia} represent the spin, flavor, and spin-flavor generators of SU(6), respectively. The generators S_c^i, T_c^a, G_c^{ia} refer to those acting upon the $N_c - 1$ core quarks, while separate SU(6) generators s^i, t^a, and g^{ia} act only on the single excited quark. Factors of N_c originate either as coefficients of operators in the Hamiltonian, or through matrix elements of those operators. An n-body operator, which acts on n quarks in a baryon state, has a coefficient of order $1/N_c^{n-1}$, reflecting the minimum number of gluon exchanges required to generate the operator in QCD. Compensating factors of N_c arise in matrix elements if sums over quark lines are coherent. For example, the unit operator 1 contributes at $O(N_c^1)$, since each core quark contributes equally in the matrix element. The core spin of the baryon S_c^2 contributes to the masses at $O(1/N_c)$, because the matrix elements of S_c^i are of $O(N_c^0)$ for baryons that have spins of order unity as $N_c \to \infty$. Similarly, matrix elements of T_c^a are $O(N_c^0)$ in the two-flavor case since the baryons considered have isospin of $O(N_c^0)$, but the operator G_c^{ia} has matrix elements on this subset of states of $O(N_c^1)$. This means that the contributions of the $O(N_c)$ quarks add incoherently in matrix elements of the operator S_c^i or T_c^a but coherently for G_c^{ia}. Thus, the full large N_c counting of the matrix element is $O(N_c^{1-n+m})$, where m is the number of coherent core quark generators. A complete operator basis for the nonstrange **70**-plet masses is shown in Table 1[3]. Index contractions are left implicit wherever they are unambiguous, and the c_i are operator coefficients. The tensor $\ell_{ij}^{(2)}$ represents the rank two tensor combination of ℓ^i and ℓ^j given by $\ell_{ij}^{(2)} = \frac{1}{2}\{\ell_i, \ell_j\} - \frac{\ell^2}{3}\delta_{ij}$.

Table 1
A complete operator basis, $\mathcal{O}_i$, $i = 1 \ldots 9$, for the nonstrange **70**-plet masses.

$c_1 \mathbf{1}$	$c_2 \ell s$	$c_3 \frac{1}{N_c} \ell^{(2)} g G_c$
$c_4 (\ell s + \frac{4}{N_c+1} \ell t G_c)$	$c_5 \frac{1}{N_c} \ell S_c$	$c_6 \frac{1}{N_c} S_c^2$
$c_7 \frac{1}{N_c} t T_c$	$c_8 \frac{1}{N_c} \ell^{(2)} s S_c$	$c_9 \frac{1}{N_c^2} \ell^i g^{ja} \{S_c^j, G_c^{ia}\}$

Operators 1, 2–3, and 4–9 have matrix elements of order N_c^1, N_c^0, and N_c^{-1}, respectively.

3. Results

Since the operator basis in Table 1 completely spans the 9-dimensional space of observables, we can solve for the c_i given the experimental data. For each baryon mass, we assume that the central value corresponds to the midpoint of the mass range quoted in the *Review of Particle Properties* [7]; we take the one standard deviation error as half of

[3]Some of these operators were considered previously in Ref. [6].

the stated range. To determine the off-diagonal mass matrix elements, we use the mixing angles extracted from the analysis of strong decays given in Ref. [5], $\theta_{N1} = 0.61 \pm 0.09$ and $\theta_{N3} = 3.04 \pm 0.15$. These values are consistent with those obtained in [8] from radiative decays. Solving for the operator coefficients, we obtain the values shown in Table 2.

Table 2
Operator coefficients in GeV, assuming the complete set of Table 1.

c_1	c_2	c_3	c_4	c_5	c_6	c_7	c_8	c_9
$+0.470$	-0.036	$+0.369$	$+0.089$	$+0.087$	$+0.418$	$+0.040$	$+0.048$	$+0.012$
± 0.017	± 0.041	± 0.208	± 0.203	± 0.157	± 0.085	± 0.074	± 0.172	± 0.673

Naively, one expects the c_i to be of comparable size. Using the value of c_1 as a point of comparison, it is clear that there are no operators with anomalously large coefficients. Thus, we find no conflict with the naive $1/N_c$ power counting rules. However, only three operators of the nine, $\mathcal{O}_1$, $\mathcal{O}_3$, and $\mathcal{O}_6$, have coefficients that are statistically distinguishable from zero! A fit including those three operators alone is shown in Table 3, and has a χ^2 per degree of freedom is 1.87. Fits involving other operator combinations are studied in Refs. [3,4]. Clearly, large N_c power counting is not sufficient by itself to explain the $\ell = 1$ baryon masses—the underlying dynamics plays a crucial role.

Table 3
Three parameter fit using operators $\mathcal{O}_1$, $\mathcal{O}_3$, and $\mathcal{O}_6$, giving χ^2/d.o.f. $= 11.19/6 = 1.87$. Masses are given in MeV, angles in radians.

	Fit	Exp.		Fit	Exp.
$\Delta(1700)$	1683	1720 ± 50	$N(1520)$	1530	1523 ± 8
$\Delta(1620)$	1683	1645 ± 30	$N(1535)$	1503	1538 ± 18
$N(1675)$	1673	1678 ± 8	θ_{N1}	0.45	0.61 ± 0.09
$N(1700)$	1725	1700 ± 50	θ_{N3}	3.04	3.04 ± 0.15
$N(1650)$	1663	1660 ± 20			

Parameters (GeV): $c_1 = 0.461 \pm 0.005$, $c_3 = 0.360 \pm 0.059$, $c_6 = 0.453 \pm 0.030$

4. Interpretation and Conclusions

We will now show that the preference in Table 2 for two nontrivial operators, $\frac{1}{N_c}\ell^{(2)} g\, G_c$ and $\frac{1}{N_c}S_c^2$, can be understood in a constituent quark model with a single pseudoscalar meson exchange, up to corrections of order $1/N_c^2$. The argument goes as follows:

The pion couples to the quark axial-vector current so that the $\overline{q}q\pi$ coupling introduces the spin-flavor structure $\sigma^i\tau^a$ on a given quark line. In addition, pion exchange respects the large N_c counting rules given in Section 2. A single pion exchange between the excited quark and a core quark is mapped to the operators $g^{ia}G_c^{ja}\ell_{ij}^{(2)}$ and $g^{ia}G_c^{ia}$, while pion exchange between two core quarks yields $G_c^{ia}G_c^{ia}$. These exhaust the possible two-body operators that have the desired spin-flavor structure. The first operator is one of the two in our preferred set. The third operator may be rewritten

$$2G_c^{ia}G_c^{ia} = C_2 \cdot 1 - \frac{1}{2}T_c^a T_c^a - \frac{1}{2}S_c^2 \tag{1}$$

where C_2 is the SU(4) quadratic Casimir for the totally symmetric core representation (the **10** of SU(4) for $N_c = 3$). Since the core wavefunction involves two spin and two flavor degrees of freedom, and is totally symmetric, it is straightforward to show that $T_c^2 = S_c^2$. Then Eq. (1) implies that one may exchange $G_c^{ia}G_c^{ia}$ in favor of the identity operator and S_c^2, the second of the two operators suggested by our fits.

The remaining operator, $g^{ia}G_c^{ia}$, is peculiar in that its matrix element between two nonstrange, mixed symmetry states is given by [3]

$$\frac{1}{N_c}\langle gG\rangle = -\frac{N_c+1}{16N_c} + \delta_{S,I}\frac{I(I+1)}{2N_c^2} \ , \tag{2}$$

which differs from the identity only at order $1/N_c^2$. Thus to order $1/N_c$, one may make the replacements

$$\{1 \ , \ g^{ia}G_c^{ja}\ell_{ij}^{(2)} \ , \ g^{ia}G_c^{ia} \ , \ G_c^{ia}G_c^{ia}\} \Rightarrow \{1 \ , \ g^{ia}G_c^{ja}\ell_{ij}^{(2)} \ , \ S_c^2\} \ . \tag{3}$$

We conclude that the operator set suggested by the data may be understood in terms of single pion exchange between quark lines. This is consistent with the interpretation of the mass spectrum advocated by Glozman and Riska [9]. Other simple models, such as single gluon exchange, do not directly select the operators suggested by our analysis and may require others that are disfavored by the data.

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